

UNIVERSITY OF KERALA
ENTRANCE TEST FOR Ph.D. (STATISTICS) ADMISSION- 2016

Time: 3 Hours

Total Marks: 100

Part-A

Answer any 10 questions. Each question carries 5 marks.

1. How do you define a research problem? Write down the objectives of research.
2. Give an account of various types of research.
3. What are the advantages and disadvantages of web pages as a source of information in comparison with printed documents?
4. What do you mean by hypothesis? What are different types of hypotheses?
5. Briefly explain various methods of collecting data.
6. Explain the basic principles of experimental designs.
7. Describe some of the important research designs used in experimental hypothesis testing in research study.
8. Mention the different types of reports, particularly pointing out the difference between a technical report and a popular report.
9. Describe various types of illustrations used in research reports.
10. Explain different types of intellectual property rights.
11. What is plagiarism? How does plagiarism infringe intellectual property rights.
12. Explain various types of sampling methods.
13. Distinguish between primary and secondary sources of scientific research.
14. Write a note on critical literature review.
15. Describe, in brief, the layout of a research report, covering all relevant points.

Part-B

Answer any 10 questions. Each question carries 5 marks.

16. Prove or disprove: "Every subset of R is Lebesgue measurable".
17. State and prove the inequality relationship between geometric multiplicity and algebraic multiplicity of a characteristic root.
18. Let X be distributed as Uniform $(0,1)$ and let $Y = \max\{X, 1-X\}$, $T = -\ln X$. Derive the distribution function of Y and density of T .
19. Let $X \sim \text{beta}(\alpha_1, \beta_1)$ and $Y \sim \text{beta}(\alpha_2, \beta_2)$ be two independent random variables. Obtain probability density function of XY , if $\alpha_2 = \alpha_1 + \beta_1$.
20. If $\underline{X} = (X_1, X_2, X_3)$ follows $N_3(0, \Sigma)$, where $\Sigma = \begin{pmatrix} 1 & 0.8 & -0.4 \\ 0.8 & 1 & -0.56 \\ -0.4 & -0.56 & 1 \end{pmatrix}$. Then find (i) conditional distribution of X_1 and X_2 given X_3 and (ii) partial correlation coefficient between X_1 and X_2 given X_3 .
21. Let X_1 and X_2 be two independent and identically distributed geometric random variables with parameter p . Obtain the conditional distribution of X_1 given $X_1 + X_2$.
22. State and prove Bayes theorem for a countable number of events.
23. State (without proof) WLLN, SLLN and CLT regarding a sequence $\{X_n, n \geq 1\}$ of independent and identically distributed random variables.
24. If X_1, X_2, \dots, X_n are independent and identically distributed positive random variables with $E(X_j^{-1}) < \infty$ for $j=1, 2, \dots, n$, then show that $E\left(\frac{X_1 + X_2 + \dots + X_m}{X_1 + X_2 + \dots + X_n}\right) = \frac{m}{n}$, for $m \leq n$.
25. Examine uses of auxiliary information in survey sampling.

26. The intra block subgroup of a confounding system in a 2^4 experiment involving the factors A, B, C and D has the treatment combinations "(1), ad, bc, abcd". Determine the other blocks of the confounding and all interactions which are confounded in the generated, confounded arrangement. If there are r repetitions of the above type of confounding, then outline the ANOVA table of the experiment.
27. Given a random sample of size n drawn from the Cauchy distribution with pdf $f(x; \mu, \sigma) = \frac{1}{\pi} \frac{\sigma}{\sigma^2 + (x - \mu)^2}$. Discuss a method of estimating μ and σ of the above distribution.
28. State (without proof): Cramer-Rao inequality, Gauss-Markov Theorem, Fisher-Neyman Factorization Theorem.
29. If $\phi(x)$ given by $\phi(x) = 1$, if $x \geq 2$; $= 0$, if $x < 2$ is a test for testing the hypothesis $H_0 : \theta = 2$ against $H_1 : \theta = 1$ based on a single observation taken from an exponential distribution with mean θ , then (i) find the probability of Type I error (ii) find the probability of Type II error & (iii) check whether ϕ is unbiased or not.
30. Explain likelihood ratio test and SPRT. Give an example for each.